Entry Task: (Motivation/WarmUp)
$1000 is deposited in an account.
We call this the *principal*: P = 1000.

1. What is 20% of $1000?

2. If 20% of the principal is added to the balance, what is the new balance?

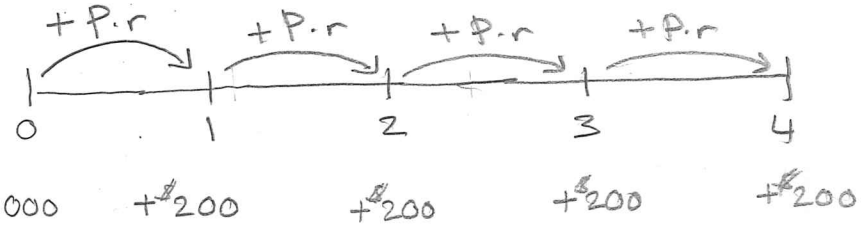
3. If 20% of the *original principal* is added to the balance at the end of each year, what is the balance at the end of 4 years?

4. If 20% of the *entire balance* is added to the balance at the end of each year, what is the balance at the end of 4 years?

[1] $\underbrace{1000}_{P} \cdot \underbrace{0.20}_{r} = \boxed{\$200}$

[2] $\underbrace{1000}_{P} + \underbrace{1000 \cdot 0.20}_{P \cdot r} = \boxed{\$1200} = \underbrace{1000}_{P} \cdot (1 + \underbrace{0.2}_{r})$ $\quad P \cdot (1+r)$

[3] "SIMPLE INTEREST"



$1000 $+200 $+200 $+200 $+200

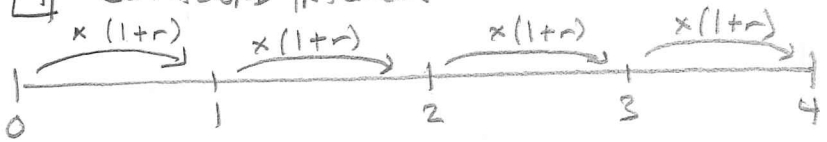
Present Value

$\underbrace{1000}_{P} + \underbrace{200 \cdot 4}_{P \cdot r \cdot t} = \boxed{\$1800}$

INTEREST = $800

Future Value

[4] "COMPOUND INTEREST"



$1000 \cdot (1+0.2) = 1200$

$1200 \cdot (1+0.2) = 1440$

$1440 \cdot (1+0.2) = 1728$

$1728 \cdot (1+0.2) = 2073.60$

FUTURE VALUE

ALSO $1000\,(1+0.2)^4 = \boxed{\$2073.60}$

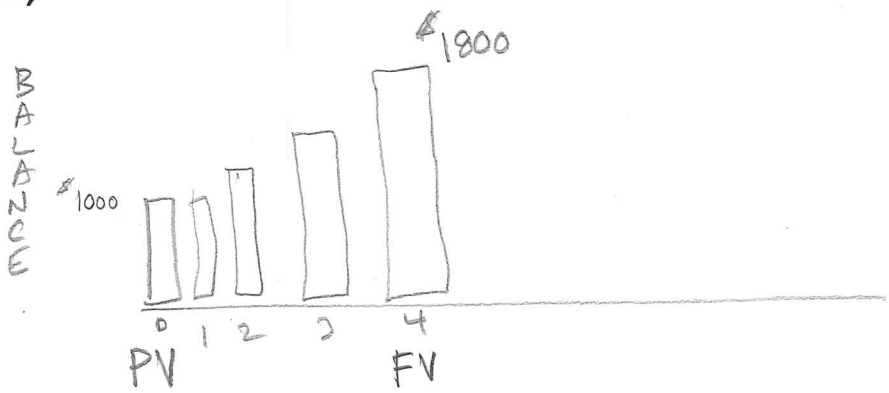
PRESENT VALUE

INTEREST = $1073.60

Quick Preview

6.1 & 6.2 Preview - Lump Sums

A *lump sum* (called the *present value*, or *principal*) is deposited into an account today.



INTEREST = FV − PV = 1800 − 1000 = 800

The account earns interest, but <u>no other deposits are made</u>.

Terminology:

P = PV = present (start) value

F = FV = future (end) value

I = total interest = F – P

r = decimal interest rate,

t = time

m = num. of comp. periods per year

n = mt = total num. of compoundings

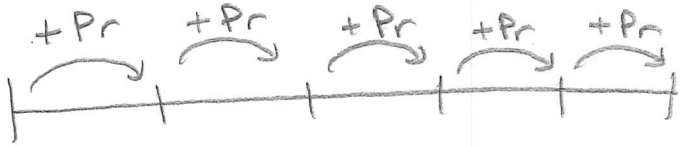
} More on this next time

Three types of lump sum accounts

1. Simple Interest (no compounding):

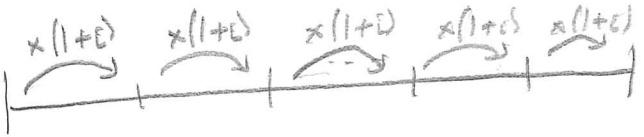
$$F = P + Prt$$

TODAY



Quick Review

Compounding: $F = P\,(1+i)^n$



Two types of compounding:

2. Compound *n* times/year:

$$F = P\left(1 + \frac{r}{m}\right)^{m\,t}$$

MORE ON THIS NEXT TIME

3. Continuous compounding:

$$F = Pe^{r\,t}$$

6.1 Simple Interest

When an account says it pays **simple interest** that means that interest is only calculated on the original principal. In other words, the same dollar amount is added to the account each year (interest is NOT compounding).

Example (Prob. 1 from homework): You invest $10,000 for 8 years into an account that pays 17% simple interest annually. Give the future value and the total interest.

SIMPLE INTEREST

$\Rightarrow F = P + Prt$

$P = 10000$

$r = 0.17$

$t = 8$

$F = 10000 + 10000 \cdot 0.17 \cdot 8$

$= 10000 + 1700 \cdot 8$

$= 10000 + 13600$

$= 23600$

$\boxed{F = \$23{,}600}$

YOU GOT ↓ — YOU PUT IN ↓

INTEREST $= 23600 - 10000$

$= \$13600$

SIDE NOTE: HW 2, 3, 4 CAN USE FRACTIONS OF A YEAR

- 9 months $\Rightarrow t = \frac{9}{12} = 0.75$ yrs
- 15 months $\Rightarrow t = \frac{15}{12} = 1.25$ yrs
- 6 months $\Rightarrow t = \frac{6}{12} = 0.5$ yrs

6.1 Miscellaneous Homework:

(all 6.1 homework is simple interest!)

7. If $5000 is invested at 9% annual simple interest, how long does it take to be worth $8150?

P = 5000 , t = ? , r = 0.09 , F = 8150

$F = P + Prt$

$8150 = 5000 + \underbrace{5000 \cdot 0.09 \cdot t}_{450}$ $\}$ -5000

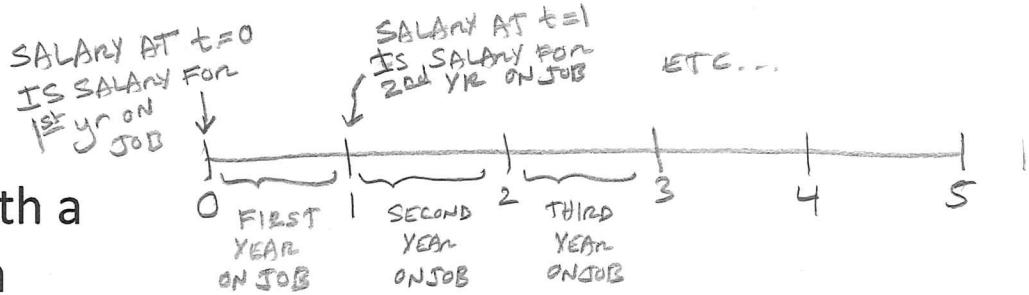
SOLVE FOR t

$3150 = 450t$

$t = \frac{3150}{450} = 7$ years

10. Suppose you are offered a job with a relatively low starting salary but with a $4000 raise for each of the next 12 years. How much more than your starting salary would you be making in the thirteenth year?

P = ?? , t = 12 , r = ?? , F = ??



WANTS TO KNOW SALARY AFTER $t = 12$ RAISES!

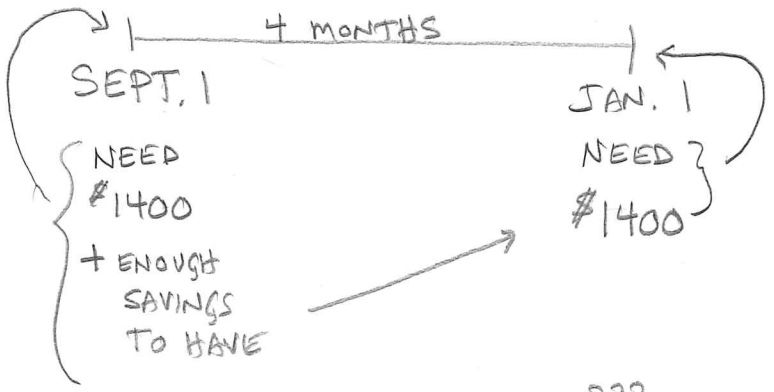
$F = P + \underbrace{Prt}_{4000}$

$F = P + 4000 \cdot 12$

$F = P + 48000$

$48,000 MORE

5. A student has a savings account earning 3% simple interest. She must pay $1400 for first-semester tuition by September 1 and $1400 for second-semester tuition by January 1. How much must she earn in the summer (by September 1) in order to pay the first-semester bill on time and still have the remainder of her summer earnings grow to $1400 between September 1 and January 1?



AMOUNT NEEDED ON SEPT. 1 $= \$1400 + P$

NEED P TO GROW TO $1400 in 4 months

$$F = P + Prt$$

$F = 1400$
$P = ?$
$r = 0.03$
$t = \frac{4}{12} = 0.\overline{3}$ yrs

$$1400 = P + P \cdot 0.03 \cdot 0.\overline{3}$$
$$1400 = P(1 + \underbrace{0.03 \cdot 0.\overline{3}}_{0.01})$$
$$1400 = P(1.01)$$
$$P = \frac{1400}{1.01}$$
$$\approx 1386.138614$$

ABOUT $1386.14

TOTAL NEEDED ON SEPT. 1 $= 1400 + 1386.\overline{14}$

$= \boxed{\$2786.\overline{14}}$

8. A retailer owes a wholesaler $500,000 due in 45 days. If the payment is 15 days late, there is a 1% penalty charge. The retailer can get a 45-day certificate of deposit (CD) paying 8% or a 60-day certificate paying 9%. Is it better to take the 45-day certificate and pay on time or to take the 60-day certificate and pay late with the penalty?
Note: P = 500000
(they have the $500,000, now!)

OPTION 1 SAVE FOR 45-DAYS & PAY ON TIME

$P = \$500{,}000$ ← COMPANY HAS THIS AND IS TRYING TO FIND OPTIMAL WAY TO MANAGE CASH!

$r = 0.08$

$t = \frac{45}{365} \approx 0.123287671$ years

$F = P + Prt$

$= 500000 + 500000 \cdot 0.08 \cdot 0.123287671$

$\approx 500000 + 4931.50685$

$\approx \$504931.51$ ← AFTER YOU PAY $500000 (NO LATE FEE)

INTEREST $= \$4931.51$ ← MONEY YOU GET FOR INVESTING & WAITING 45 DAYS TO PAY!

OPTION 2 SAVE FOR 60-DAYS, BUT PAY A 1% LATE FEE

NOTE: WILL HAVE TO PAY

$500000 + \underbrace{0.01 \cdot 500000}_{5000 \text{ LATE FEE}}$

$= 505000$

$P = \$500000$

$r = 0.09$

$t = \frac{60}{365} \approx 0.164383562$ years

$F = P + Prt$

$= 500000 + 500000 \cdot 0.09 \cdot 0.164383562$

$= 500000 + 7397.26074$

≈ 507397.26

MORE INTEREST EARNED ($\$7397.26$)

BUT HAVE TO PAY 505000

SO MONEY LEFT OVER $= 507397.26 - 505000$

$= \$2397.26$

NOT AS MUCH AS OPTION 1!!

CHOOSE OPTION 1, PAY ON TIME!